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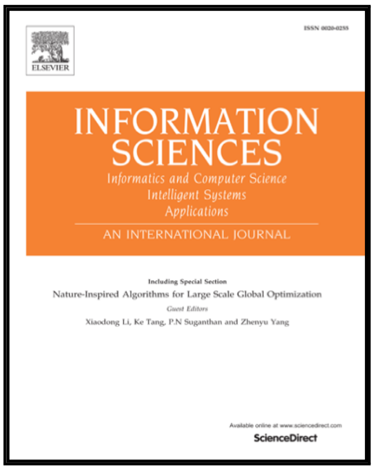

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Highlights

- A reliable location allocation model for hazardous materials is formulated.
- Scenario of depot disruption is considered.
- Numerical examples show that the proposed model leads to 8.33

Reliable location allocation for hazardous materials

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Abstract

Decision-making on location allocation has gained considerable attention since it involves strategic and operational policies with mid-term and long-term effects. In this paper, a reliable location allocation mechanism in the context of hazardous materials is proposed, considering that the depots are subject to the risk of disruption that may be caused by many factors, including hazardous materials depot accident, depot maintenance upgrade, equipment fault, and power outage. According to the characteristics of hazardous materials, this study aims to minimize the systemic risk of storage and transportation under budget constraints. The problem is formulated as an integer linear programming model to simultaneously determine: (i) the optimal depot locations; (ii) the amount of hazardous materials stored in each located depot; (iii) the optimal allocation (transportation) plans; and (iv) the contingency plans for depot disruption. Numerical examples demonstrate that the proposed modeling method leads to 8.33% risk reduction and 1.92% cost savings compared with traditional location allocation without disruption consideration. This reveals the necessity and importance of taking reliability into account and making contingency plans in disruption scenarios regarding hazardous materials location allocation decisions.

Keywords: Location allocation; hazardous materials; storage risk; transportation risk; depot disruption

1 Introduction

Hazardous materials (hazmats in short) are products that are poisonous, radioactive, explosive, corrosive, flammable, or infectious, mainly including fuel oil, liquified natural gas, dynamite, strong acid, strong alkali and so on. Although hazmats are associated with these dangerous characteristics, most of them are of great importance as a fundamental part of both daily lives and industrial production. However, in recent years, hazmats accidents that occured mainly in storage and transportation processes have caused catastrophic losses to humans and environment all over the world. According to the accident report produced by the China Chemical Safety Association, there have been 1012 dangerous chemical accidents in China from 2011 to 2016, which had caused the death of 1160 people. For instance, on July 19, 2014, an extraordinary serious accident involving hazmats happened in the course of transportation in Shaoyang City of Hunan Province, resulting in 54 deaths and 53 million RMB direct economic losses. Even more shocking is the explosion of hazmats depots in Tianjin port, occured in August 12, 2015, which killed 165 people and caused direct economic losses amount to nearly 7 billion RMB and immeasurable indirect economic losses. Such huge casualties and loss of properties are related to the highly risky location of the hazmats depot, which is only 600 meters away from a large community of 5600 residents.

In the design of hazmats logistics systems, location and allocation are two crucial issues which can help reduce the risk to a great extent. This is because both storage risk and transportation risk are closely related to

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depot location and customer allocation. On the one hand, depot locating chooses the optimal depots from the candidates set and has a direct impact on storage risk and an indirect impact on transportation risk. On the other hand, according to the located depots, customer allocating makes the optimal allocation (transportation) plans and decides the amount of hazmats stored in each depot, which has a direct impact on transportation risk and an indirect impact on storage risk. Therefore, in this paper, we study integrated location allocation in the context of hazmats with the objective of minimizing the total storage and transportation risk.

Location allocation, as a problem originally introduced by Cooper (1963), has been widely investigated for formulating various practical issues. Many mathematical models, including integer programming, dynamic programming, stochastic programming, fuzzy programming, and fuzzy random programming have been proposed for this problem in the existing literature. Along with the relevant theoretical developments, approaches to location allocation have been applied to many practical fields, such as supply chain design (Chen et al. 2015), traffic network design (Zockaie et al. 2018), storage and allocation of post-disaster relief materials (Duhamel et al. 2016), and spatial planning of water and/or energy access networks (Gokbayrak and Kocaman 2017). However, to the best of our knowledge, applications of location allocation in the context of hazmats logistics have not yet been addressed. In hazmats logistics, location and allocation are generally handled separately due to their complexity (Campbell and O'Kelly 2012; Xu et al. 2018), which will inevitably lead to suboptimal results (Wesolowsky and Truscott 1975). In this study, as the first attempt to bridge the gap between location allocation and hazmats logistics, we will construct an integer programming model for making joint location allocation decisions.

For lowering the storage risk, hazmats storage facilities have very strict safety standards. As such depot unavailability may occur over a certain period of time because of depot maintenance upgrade or equipment fault rectification. In addition, the power outage, hazmats incidents, or changes in ownership can also lead to the unavailability of depots. If any of such unavailabilities of depots were not considered before making the location decision, it could cause great losses to the company concerned. Shortage of supply due to depot disruption not only adversely affects the reputation and image of the company but also causes a large account of penalty charges. Besides, re-allocation of customers may result in excessive transportation risks and costs. Whilst numerous studies taking reliability into consideration have been conducted in supply chains and logistics management (Mohammadi and Tavakkoli-Moghaddam 2016; Wang et al. 2018), investigations into reliable location allocation in the context of hazmats logistics remains an under-studied topic.

Previous studies on hazmats logistics mainly focused on minimizing transportation risk without reliablity consideration (Du et al. 2016). However, decision-making on location allocation should also take the reliablity and the storage risk into account, since it involves strategic and operational policies with mid-term and long-term effects. On one hand, decision-making on location without depot disruption consideration might increase the risks and costs of company concerned since depots are subject to the risk of disruption. On the other hand, the storage risk and the transportation risk should be optimized simultaneously since the hazmats accidents occur mainly in storage and transportation processes. Therefore, this study minimizes the systemic risk which includes not only the storage risk but also the transportation risk in all scenarios of depot disruption. What is more, it is necessary to optimize the supply amount to every customer in the scenario of depot disruption, since depot disruption usually results in the shortage of supply. Furthermore, from the perspective of operations management, it is of scientific significance to make an contingency plan for each scenario of depot disruption so as to minimize the risks and potential economic losses in the event of depot disruption.

Inspired by these observations mentioned above, we proposed a noval reliable location allocation mechanism in the context of hazmats. The contribution of this paper is first to formulate a reliable location allocation model which takes all scenarios of depot disruption into consideration to integrally optimize location, allocation and contingency plans. A major difference of our study with previous hazmats logistics studies is that we focus on minimizing the systemic risk, given that decision-making on location allocation involves strategic and operational policies with mid-term and long-term effects. Moreover, making the contingency plans for every scenario of depot disruption can be an effective mechanism for minimizing the risks and economic losses caused by supply shortage due to depot disruption. Numerical experiments reveals the necessity and importance of taking reliability into account and shows the superiority of our proposed model. What is more, the stability of this model is illustrated by implementing two statistical tests. Finally, some management insights are obtained from a case studies in real-world.

The remainder of this paper is structured as follows. Section 2 reviews the related literature. Section 3 formulates the reliable location allocation problem as an integer linear programming model. The global optimal solution of this problem is obtained by running the Lingo software. Section 4 presents illustrative examples to verify the effectiveness of the proposed model, followed by several comparative experiments with other similar approaches. A case study of our approach is summarized in Section 5. Finally, Section 6 concludes the paper and briefly discusses potential future research directions.

2 Literature review

In this paper, we concentrate on formulating the reliable location allocation model for hazmats logistics. In what follows, we review related literatures on location allocation problem, reliable facility location problem, and hazmats logistics management.

2.1 Location allocation problem

Depot location is a crucial issue in logistics systems design due to its wide applications in real life problems. Numerous studies (Weber 1957; Gentili and Mirchandani 2018; He et al. 2018) have been developed to address this issue. Dealing with it together with other important problems such as allocation, routing, and inventory has also been considered, as making location decisions independently may lead to suboptimal results. These problems are respectively called location allocation problem (LAP) (Brimberg and Love 1998; Zarrinpoor et al. 2017; Mogale et al. 2018), location routing problem (Wei et al. 2015; Gianessi et al. 2016; Hof et al. 2017), and location inventory problem (Karmarkar 1981; Ozsen et al. 2009; Zhang and Unnikrishnan 2016). The present study focuses on LAP, with the relevant literature outlined below.

Over the last decade, LAP has been extensively developed and applied in various fields such as health-care systems, supply chain design, telecommunication networks. For example, considering the randomness of customer arrival in conjunction with the issue of facility congestion, Hajipour et al. (2016) introduced queuing theory into the study of congested facility location allocation which can be exploited to support health-care. A multi-objective facility location allocation model with congested facilities was constructed to minimize the travel and waiting times, the construction cost, and the maximum idle probability using a Pareto-based meta-heuristic. Another example is the tri-level location allocation model developed by Fard and Hajaghaei-Keshteli (2018); they proposed a nested metaheuristic approach to designing the supply chain network including distribution centers, customer zones, and recover centers. Also, to perform cellular network design in support of emergency notification, Akella et al. (2005) formulated a mixed integer programming technique to decide on the location of base stations and the allocation of channels. Four different greedy heuristics were proposed to obtain high quality solutions efficiently and a Lagrangean heuristic is built to improve the optimality gap.

Moreover, LAP has also achieved good application results in the following fields: car-sharing networks, public security systems, and emergency services. For instance, Correia and Antunes (2012) investigated the problem of depot locating and vehicle allocating in a one-way car-sharing network from the perspective of maximizing the profits of the car-sharing organization. They proposed an optimization approach for depot location and trip selection based on mixed-integer programming models. To prevent or reduce traffic offences, road accidents and traffic congestion, Adler et al. (2014) formulated the location allocation problem using four integer linear programs to jointly address the location and allocation of traffic police routine patrol vehicles. Sherali et al. (2008) developed another location allocation model for emergency services and designed an exact implicit enumeration algorithm to determine the optimal locations of shelters, making the evacuation plan with minimum evacuation time. Although LAP has been widely researched and applied, work on LAP for hazmats logistics remains an under studied topic according to our survey of the literature.

2.2 Reliable facility location problem

In the wide spectrum of facility location problems there exist approaches which assume that the facilities are always available. Based on this assumption, location studies have been rapidly developed over the past years. However, in the real world, facilities are subject to the risk of probabilistic disruptions. Decisions without considering facility disruption could incur significant economic losses. Therefore, reliable facility location problem has attracted more and more attention in recent years. In particular, Snyder and Daskin (2005) proposed such a model to minimize the weighted sum of operating costs and expected failure costs, and presented a Lagrangian relaxation algorithm to resolve this model. Considering the probability of site-dependent disruption and customer reallocation, Cui et al. (2010) proposed a discrete model and a continuum approximation to minimize the sum of setup costs and expected transportation costs in normal and disruption scenarios. These two studies presented classical approaches to reliable facility location problem. However, some practical situations are not sufficiently investigated. For example, the assumption that the facilities are uncapacitated may be unrealistic in practice, and the customer demand generally can not be satisfied in disruption scenarios.

Recently, An et al. (2014) introduced a two-stage robust p-median reliable model to minimize the weighted sum of the operation costs in the normal disruption-free scenario and in the worst disruptive scenarios. In the first stage, locations and capacities are determined; in the second stage, allocation (transportation) decisions are adjusted after demand is given. A new column-and-constraint generation method, which is enhanced by improvement strategies based on structural properties, is designed to resolve the model. The quality of the solutions strongly depends on the high accuracy of the random representation of the parameter values. Unfortunately, such an accuracy is not usually available. Thus, Álvarez-Miranda et al. (2015) integrated robust optimization with the two stage reliable location model to develop a recoverable robust facility location allocation mechanism. Although the two-stage robust optimization considered facility capacities and demand changes due to disruptions, it neglected that the disruption probability of each depot would also affect the optimization results.

Most recently, reliable facility location problem has been applied in various fields such as post-disaster relief (Hamidi et al. 2017), networks design (Rostami et al. 2018), and earthquake preparedness (Paul & Wang 2019). Nevertheless, all these studies led to a reliable location model from the perspective of minimizing costs. Having considered the characteristics of hazmats, this study will construct a reliable location allocation model to minimize the systemic risk. What is more, we will make contingency plans for each scenario of depot disruption.

2.3 Hazmats logistics management

Existing literatures on hazmats logistics management can be divided into different streams (Fontaine & Minner 2018): risk assessment (Cordeiro et al. 2016), routing (Hu et al. 2017), facility location combined with routing (Romero et al. 2016) and network design (Taslimi et al. 2017). In recent studies, Wei et al. (2015) considered fuzzy-randomness in location-scheduling programming for hazmats transportation, by assuming transportation risks as fuzzy random variables influenced by variations of time periods and road conditions. They developed a time-dependent location-scheduling programming method to optimize the location of depots and the scheduling of vehicles within a time-dependent traffic network. To minimize the potential consequences on the most exposed vulnerable places such as schools, hospitals and senior citizens' residences, Bronfman et al. (2015) constructed a maximin hazmats routing model and presented an optimal heuristic algorithm to resolve it. In view of the fuzziness of transportation risk, Du et al. (2017) formulated a fuzzy bilevel programming model where fuzziness was exploited to measure and minimize the risk during the transportation with the upper level assigning customers to depots and the lower determining the optimal routing solution. Considering the traffic restrictions on certain inter-city roads for hazmats vehicles, Hu et al. (2018) presented a multi-objective location-routing model to find the optimal routes in hazmats logistics. These models for hazmats logistics can all obtain the local optimal solutions, through the application of heuristics such as fuzzy simulation-based genetic algorithm, greedy search based adaptive hybrid particle swarm optimization algorithm, and improved genetic algorithm with two types of genes. Despite these efforts, to the best of our knowledge, there has not been any reliable location study reported that considered depot disruption in location allocation programming for hazmats logistics.

Based on the literature outlined above, we can see that many studies taking reliability into consideration in supply chain management have been conducted. Also, facility location and transportation of hazmats has gained considerable attention due to the potential disastrous consequences caused by hazmats accidents. However, to our best knowledge, location allocation together with reliability study has not been considered in the context of hazmats logistics, which forms the focus of this study.

3 Reliable location allocation modeling

In this section, we first describe the problem of reliable location allocation for hazmats. Then we introduce the notations that will be used throughout this paper. Finally, we formulate the reliable location allocation (RLA) model and the traditional location allocation (LA) model which does not address the issue of disruption for hazmats logistics. These two models will be taken for comparative experiment evaluations in the next section.

3.1 Problem description

In real-world operations, hazmats depots are subject to disruption caused by hazmats incidents, depot maintenance upgrade, equipment fault, power outage and so on. Since depot location is a long-term decision that cannot be changed shortly, the location decision without disruption consideration is likely to bring more risks and losses. Therefore, this study integrally considers the scenarios of both depot availability and depot disruption to make reliable location allocation decisions for hazmats logistics. Furthermore, in order to minimize the risks and losses of the company after a certain depot disruption, the contingency plans that should be developed in advance are also considered here.

According to the typical characteristics of hazmats depots, depot disruptions are not generally correlated, and the probability of simultaneous disruption of two or more depots is very small. This is because the maintenance and upgrade of depots and equipments are usually carried out in turn, depots rarely have a power failure at the same time thanks to the different locations of depots, and accidents at different depots rarely happen simultaneously. Therefore, in the process of modeling, this paper does not consider the scenarios that two or more depots are disrupted simultaneously and makes contingency plans only for each scenario with one depot disruption.

The purpose of this study is to derive a mechanism that helps make location allocation decision for small and medium-sized companies. In reality, small and medium-sized companies generally rent rather than build hazmats depots, since renting a depot can avoid capital investment and associated financial risk. Hence, a location here means that which given candidate depot should be rented. In this study, each vehicle is assumed to be able to only service one customer at a time, since the delivery times for different customers are different. It is also assumed that each customer can only receive service from one depot in any scenario, so that the company can readily manage the delivery. When all rented depots are available, the regular and usual demand of every customer must be satisfied. Even when one rented depot is disrupted, the minimal demand of each customer must be satisfied by the other available depots. At the same time, any shortage of supplies caused by depot disruption will be fined.

According to the characteristics of hazmats, this work aims to minimize the systemic risk of storage and transportation under given budget constraints. The objective is to simultaneously determine: (1) the optimal depot locations; (2) the amount of hazmats stored in each located depot; (3) the optimal allocation (transportation) plans; and (4) the contingency plans for depot disruption. Before establishing the optimization model, we make the following assumption:

Assumption. *Hazmats stored in the disrupted depot can not be reused.*

To understand the problem better, an illustrative example with 6 depots and 10 customers is given herein. The parameters of depots and customers are shown in Figure 1(a). The cuboids denote the candidate depots,

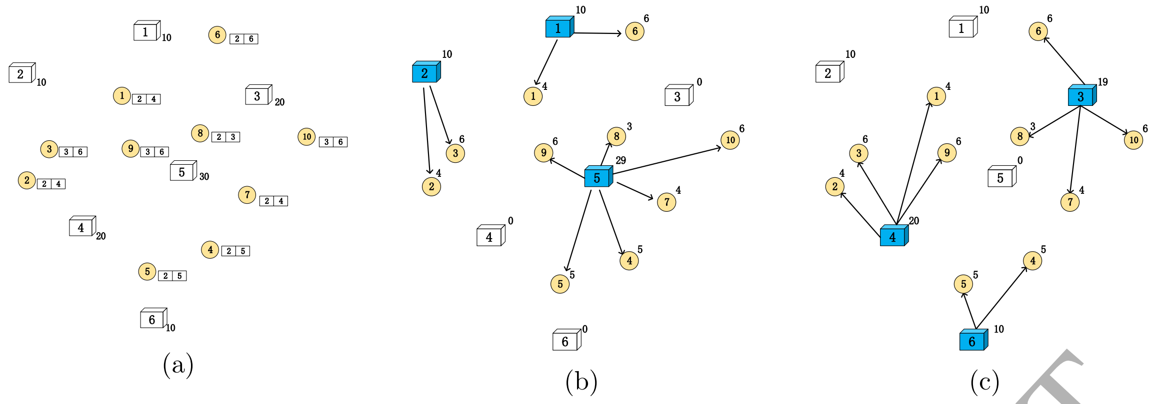

Fig. 1. Examples of location allocation decision without disruption consideration.

and the yellow circles denote the customers. The numbers labeled inside the depots and customers are their corresponding indices. The number in the lower right corner of each depot is the maximum capacity, and the numbers in the lower right corner of each customer are, from left to right, the minimum demand and the regular demand. Suppose that one case for location allocation decision without disruption consideration is given as shown in Figure 1(b). The blue cuboids denote the rented depots. The number in the upper right corner of each depot is the amount of hazmats stored in this depot, and the number in the upper right corner of each customer is the supply amount to this customer. Obviously, depots 1, 2 and 5 are rented from a set of six candidate depots. However, once depot 5 is disrupted, the minimum demands of all customers cannot be satisfied by the hazmats stored in the available depots. This will not only bring great financial losses, but also will damage the reputation of the company. Suppose that another case for location allocation decision without disruption consideration is shown in Figure 1(c). In this case, depots 3, 4 and 6 are rented from the six candidate depots. Obviously, the minimum demand by each customer can be satisfied with the hazmats stored in the available depots when one rented depot is disrupted. However, when depot 3 or 4 is disrupted the transportation cost will increase significantly. This is because most customers are far away from depot 6.

As indicated previously, in this study, we aim to minimize the systemic risk of storage and transportation under given budget constraints. Therefore, location allocation decision will be made by considering all the scenarios of depot being normal and depot with disruption as a whole. Furthermore, the contingency plans should be developed in advance. The location allocation decision with disruption consideration and contingency plans are illustrated in Figure 2. The green cuboids denote the disrupted depots. Obviously, depots 2, 3 and 4 are rented from the six candidate depots. The location allocation decision in the scenario where all location depots are available is shown in Figure 2(a). In this scenario, the regular demand of every customer must be satisfied. For example, 10 tons of hazmats are stored in depot 2, and the supply amounts to customers 1 and 3 from depot 2 are 4 and 6 tons, respectively. contingency plans are shown in Figures 2(b), 2(c) and 2(d). In each scenario of depot disruption, the supply amount from the available depots to each customer must be greater or equal to the customer's minimal demand. For example, the contingency plan for the scenario where depot 2 is disrupted is illustrated in Figure 2(b). In this scenario, customer 1, whose minimum demand is 2 tons, is supplied with 2 tons by depot 3, and customer 3, whose minimum demand is 3 tons, is supplied with 4 tons by depot 4.

3.2 Notations

To support the understanding of this work, this subsection lists all indices, parameters, and decision variables to be employed in the following modeling and solution processes.

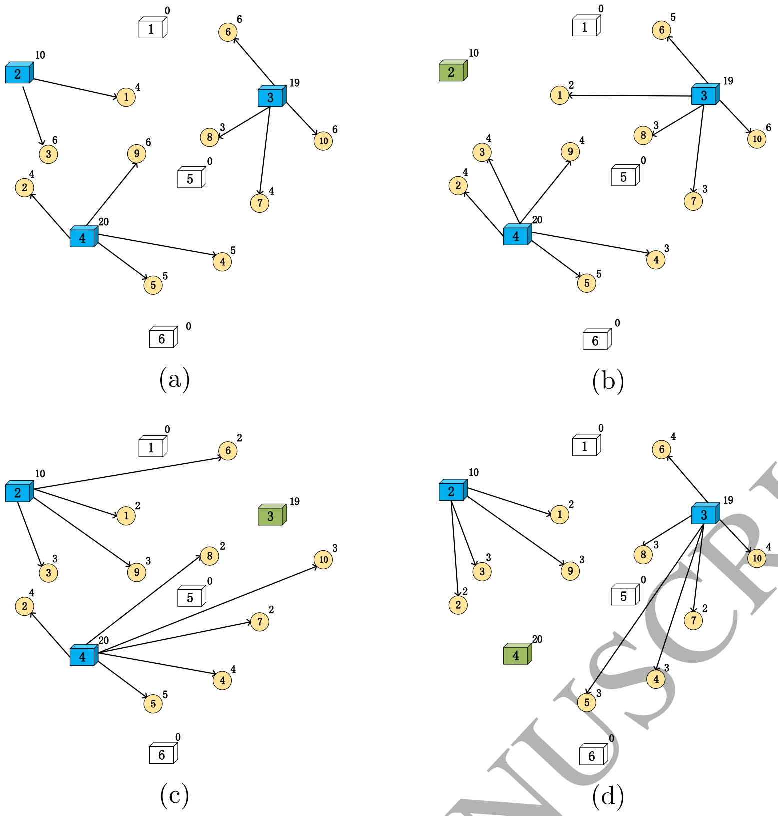

Fig. 2. Examples of location allocation decision with disruption consideration and contingency plans.

Indices and parameters

I	depot set;
i	depot index, $i = 1, 2, \cdots, \|I\|$;
J	customer set;
j	customer index, $j = 1, 2, \cdots, \|J\|$;
S	set of depot disruption scenarios;
s	scenario index, $s = 0, 1, 2, \cdots, \|S\|$;
d_j	regular demand of customer j, $j \in J$;
$\underline{d}_j$	minimum demand of customer j, $j \in J$;
c_i	maximum capacity of depot i, $i \in I$;
f_i	unit inventory cost of depot i, $i \in I$;
h_i	unit inventory risk of depot i, $i \in I$;
t_i	rental cost of depot i, $i \in I$;
l_{ij}	unit transportation cost from depot i to customer j, $i \in I$, $j \in J$;
r_{ij}	unit transportation risk from depot i to customer j, $i \in I$, $j \in J$;
k_j	shortage penalty coefficient of customer j, $j \in J$;
p_s	occurrance probability of scenario s, $s \in S$;
G	total budget.

Decision variables

$$x_i = \begin{cases} 1, & \text{if candidate depot } i \text{ is rented, } i \in I \\ 0, & \text{otherwise;} \end{cases}$$

$$y_{ij}^s = \begin{cases} 1, & \text{if customer } j \text{ is supplied by depot } i \text{ in scenario } s,\ i \in I,\ j \in J,\ s \in S \\ 0, & \text{otherwise;} \end{cases}$$

z_i amount of hazmats stored in depot i, $i \in I$;

w_{ij}^s supply amount to customer j from depot i in scenario s, $i \in I$, $j \in J$, $s \in S$.

Remark 1. *Scenario s denotes that the sth depot is disrupted, and others work well. In particular, scenario 0 indicates that all depots are available.*

Risk, as a very important factor in hazmats logistics, is a measure of accident occurrence probability and the consequence of an accident. Below is the risk model which is commonly used in dealing with hazmats logistics problems (Batta and Chiu 1988):

$$r_{ij} = p_{ij} \times \kappa_{ij} \times \tau_{ij}$$

where p_{ij} denotes the occurrence probability of an accident, κ_{ij} means the affected area of the accident, and τ_{ij} stands for the average population density. In this paper, unit transportation risk from a depot to a retailer (customer) and unit inventory risk of the depot can be calculated from this risk model.

3.3 Modeling

Companies may face significant risks and losses, if depot disruption is not taken into account in advance when deciding on depot location. Therefore, the objective of this investigation is to minimize the systemic risk which includes both inventory risk in depots and the expectation of transportation risks between depots and customers in all scenarios. The mathematical model is thus, formulated as follows:

$$\min \sum_{i \in I} \left[h_i z_i + \sum_{s \in S} \sum_{j \in J} p_s r_{ij} w_{ij}^s \right]. \tag{1}$$

Constraint (2) below is the cost constraint which comprises from left to right, the inventory costs in depots, the expectation of transportation costs in all scenarios, the expectation of penalty costs due to shortage in all scenarios of depot disruption, and the rental costs of depots:

$$\sum_{i \in I} f_i z_i + \sum_{s \in S} \sum_{i \in I} \sum_{j \in J} p_s l_{ij} w_{ij}^s + \sum_{s \in S \setminus \{0\}} \sum_{j \in J} p_s k_j (d_j x_s - \sum_{i \in I} w_{ij}^s) + \sum_{i \in I} t_i x_i \leq G. \tag{2}$$

Since the disruption of unrented depot does not have any impact on systemic risk and cost, both transportation risk and transportation cost in the scenario where an unrented depot is disrupted should be zero. Therefore, we stipulate that no customer can be supplied and there is not any penalty cost in this scenario. If the left part of the inequality sign in constraint (2) is used as the objective function, we can obtain the minimum of systemic costs. The total budget G is set to this value with a certain percentage increase that is deemed acceptable by the company. In hazmats logistics, depot rental is a one-time investment, however, transportation cost will be produced in every transport process. In addition, the primary intention of renting a depot is to store hazmats for the follow-up transportation tasks. Therefore, in this study, the yearly rental of a depot is amortized into a fixed start-up cost of each transportation task. For example, a depot will cost 500000 RMB to rent for one year and the times of transportation are 50 within that year. After amortizing 50 transportation tasks, the rental cost of this depot becomes 10000 RMB.

Constraint (3) denotes that the amount of hazmats stored in all depots is equal to the sum of all customers' regular demands:

$$\sum_{i \in I} z_i = \sum_{j \in J} d_j. \tag{3}$$

Constraint (4) guarantees that hazmats are only stored in rented depots, and the storage amount of each depot can not exceed its maximum capacity, such that

$$z_i \leq c_i x_i, \quad \forall i \in I. \tag{4}$$

Constraint (5) indicates that each customer can only be supplied by at most one depot in any scenario:

$$\sum_{i \in I} y_{ij}^s \leq 1, \quad \forall j \in J, \quad \forall s \in S. \tag{5}$$

Constraint (6) ensures that any customer can not be supplied in each scenario where the unrented depot is disrupted, namely

$$\sum_{i \in I} \sum_{j \in J} y_{ij}^s \leq |J| x_s, \quad \forall s \in S \setminus \{0\}. \tag{6}$$

Constraint (7) states that each depot can only supply to the customers it services, and the supply amount to any customer can not exceed its maximum capacity in any scenario:

$$w_{ij}^s \leq c_i y_{ij}^s, \quad \forall i \in I, \quad \forall j \in J, \quad \forall s \in S. \tag{7}$$

Constraint (8) denotes that the depot can not supply to any customer when it is disrupted, that is

$$\sum_{j \in J} w_{ij}^i = 0, \quad \forall i \in I. \tag{8}$$

Constraint (9) guarantees that every customer's regular demand must be satisfied in the scenario where all rented depots are available:

$$\sum_{i \in I} w_{ij}^0 = d_j, \quad \forall j \in J. \tag{9}$$

Constraint (10) assures that the supply amount from the available depots to each customer must be greater or equal to the customer's minimal demand in each scenario where the rented depot is disrupted, such that

$$\underline{d}_j x_s \leq \sum_{i \in I} w_{ij}^s, \quad \forall j \in J, \quad \forall s \in S \setminus \{0\}. \tag{10}$$

Constraint (11) requires that the supply amount to every customer is less than or equal to its regular demand in each scenario where the rented depot is disrupted, and that the supply amount to any customer is zero in each scenario where the unrented depot is disrupted:

$$\sum_{i \in I} w_{ij}^s \leq d_j x_s, \quad \forall j \in J, \quad \forall s \in S \setminus \{0\}. \tag{11}$$

Constraint (12) dictates that the total supply amount of each depot can not exceed its storage amount in any scenario, that is

$$\sum_{j \in J} w_{ij}^s \leq z_i, \quad \forall i \in I, \quad \forall s \in S. \tag{12}$$

Since hazmats stored in the disrupted depot can not be reused, the total amount of hazmats that can be supplied to the customers in all scenarios is:

$$\sum_{j \in J} d_j + \sum_{i \in I} x_i (\sum_{j \in J} d_j - z_i) = \sum_{j \in J} d_j + \sum_{i \in I} x_i \sum_{j \in J} d_j - \sum_{j \in J} d_j = \sum_{i \in I} x_i \sum_{j \in J} d_j,$$

constraint (13) guarantees that the hazmats which can be supplied to customers in all scenarios are all supplied to the customers, as follows

$$\sum_{s \in S} \sum_{i \in I} \sum_{j \in J} w_{ij}^s = \sum_{i \in I} x_i \sum_{j \in J} d_j. \tag{13}$$

Constraint (14) depicts the nature of the decision variables, as follows

$$x_i, y_{ij}^s \in \{0,1\}, \quad z_i, w_{ij}^s \in N, \quad \forall i \in I, \quad \forall j \in J, \quad \forall s \in S. \tag{14}$$

Summarizing the above, the following RLA model can be formulated:

$$\begin{cases} \min & \sum_{i\in I}\left[h_i z_i + \sum_{s\in S}\sum_{j\in J} p_s r_{ij} w_{ij}^s\right] \\ \text{s.t.} & \text{Constraints } (2)-(14). \end{cases} \tag{15}$$

Remark 2. *If reliability is not considered, we can develop a traditional LA model whose decision variables are as follows:*

$x_i = \begin{cases} 1, & \text{if candidate depot } i \text{ is rented, } i \in I \\ 0, & \text{otherwise;} \end{cases}$

$y_{ij} = \begin{cases} 1, & \text{if customer } j \text{ is supplied by depot } i, \ i \in I, \ j \in J \\ 0, & \text{otherwise;} \end{cases}$

z_i amount of hazmats stored in depot i, $i \in I$;

w_{ij} supply amount to customer j from depot i, $i \in I$, $j \in J$.

However, if we use the traditional LA model, the minimum demand of customers can not generally be satisfied in any scenario involving depot disruption. This will damage the reputation of company and cause financial losses. To solve this problem, we add constrain (16) such that

$$\sum_{i\in I} z_i - z_i \geq \sum_{j\in J} \underline{d}_j, \quad \forall i \in I. \tag{16}$$

The LA model can then be readily developed such that

$$\begin{cases} \min & \sum_{i\in I}\left[h_i z_i + \sum_{j\in J} r_{ij} w_{ij}\right] \\ \text{s.t.} & \sum_{i\in I} f_i z_i + \sum_{i\in I}\sum_{j\in J} l_{ij} w_{ij} + \sum_{i\in I} t_i x_i \leq G \\ & \sum_{i\in I} z_i = \sum_{j\in J} d_j \\ & z_i \leq c_i x_i, \quad \forall i \in I \\ & \sum_{i\in I} y_{ij} = 1, \quad \forall j \in J \\ & w_{ij} \leq c_i y_{ij}, \quad \forall i \in I, \quad \forall j \in J \\ & \sum_{i\in I} w_{ij} = d_j, \quad \forall j \in J \\ & \sum_{j\in J} w_{ij} = z_i, \quad \forall i \in I \\ & \sum_{i\in I} z_i - z_i \geq \sum_{j\in J} \underline{d}_j, \quad \forall i \in I \\ & x_i, y_i \in \{0,1\}, \quad z_i, w_{ij} \in N, \quad \forall i \in I, \quad \forall j \in J. \end{cases} \tag{17}$$

The LA model does not take into account depot disruption when making location allocation decisions. Therefore, once a depot is disrupted, the scheme of allocation (transportation) carried out by the available depots needs *to be re-planned. Obviously, this would increase the systemic risk and cost, since any new allocation scheme is based on the located depots that have been chosen without reliability consideration. However, the systemic risk (i.e., the inventory risk in depots and the expectation of transportation risks between depots and customers in all scenarios) is more important for making a long-term decision. By contrast, the RLA model not only takes all scenarios of depot disruption into account to minimize the systemic risk but also makes the contingency plans in advance.*

Both RLA and LA are integer linear programming models. In RLA, the number of variables is $2i(1+j+i*j)$, and the number of constraints is $i(3i*j+5j+5)+3$. For instance, if there are ten candidate depots and fifty customers, the number of variables and constraints is respectively 11020 and 17553. The number of variables and that of the constraints are both within the computable range entailed by the Lingo software. In reality, ten candidate depots and fifty customers are sufficient for small and medium-sized companies. Therefore, the RLA model can be resolved to optimality by Lingo software. Since the number of variables and that of constraints in LA are obviously less than their counterparts within the RLA model, LA can also obtain the global optimal solution using Lingo software.

4 Numerical experiments

In this section, We first present the risk model and describe the data set. Then comparison experiments of RLA model and LA model are carried out to reveal the necessity and importance of taking reliability into account. Finally, we compare with other similar approaches of reliable facility location problem to show the superiority of our proposed model. What is more, the stability of this model is illustrated by implementing two statistical tests.

Since hazmats depots are subject to the risk of disruption that may be caused by many factors, decision-making on location allocation without disruption consideration could incur significant economic losses. In this experiment, we apply the proposed RLA model to help a company to rent certain depots from the candidate depot set and design an allocation plan. According to the characteristics of hazmats, RLA model aims to minimize the systemic risk under the budget constraint.

Although risk minimization is the primary target in hazmats logistics management, budget must also be considered in order to maintain the competitiveness of company. In this experiment, the 110 % of the minimum systemic cost without risk consideration is taken as the total budget G. The locations and attributes of the candidate depots and the retailers are shown in Figure 3, where the orange cuboids denote the depots and the yellow circles denote the retailers. The numbers labeled inside the depots are their corresponding indices. The numbers in the right of each depot are, from top to bottom, the probability of each disruption scenario and the maximum capacity. The rental cost of each depot are respectively 7500, 7500, 8000, 8000, 7500 RMB. The numbers labeled inside the retailers are their regular demand. The numbers in the right of each retailer are, from top to bottom, the minimum demand and the shortage penalty coefficient. The numbers in the left of each depot are, from top to bottom, the unit inventory risk and the unit inventory cost. The unit transportation cost from a depot to a retailer is given in Table 1, and the unit transportation risk is listed in Table 2.

Example 1. *The global optimal solutions of RLA model obtained right away by executing the Lingo software are as follows. Depots 2, 3 and 4 are determined to rent, and the corresponding amounts of hazmats stored in these depots are 48, 80 and 66 ton, respectively. The systemic risk involves 31569 people, which means that 31569 people would be affected if hazmats incident incurred, with the systemic cost being 74562 RMB. The global optimal solutions are shown in Figure 4, in which the blue cuboids denote the rented depots, the green cuboids denote the depots disrupted, the yellow circles denote retailers, and the numbers labeled inside the retailers are their amount supplied. In the scenario of all depots being available, every retailer's regular demand can be satisfied. The allocation scheme for depots and retailers is shown in Figure 4 (a). In particular, if depot 1 is disrupted, the contingency plan is shown in Figure 4 (b). If depot 3 is disrupted, the contingency plan is shown in Figure 4 (c). If however, depot 4 is disrupted, the contingency plan is shown in Figure 4 (d).*

Table 1. Unit transportation cost (RMB) from depot to retailer.

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
D1	122	171	410	401	333	387	428	90	324	315
D2	121	285	352	330	442	267	348	89	406	429
D3	432	522	225	63	95	176	243	477	225	198
D4	230	108	414	302	243	257	432	315	140	180
D5	120	86	467	332	238	265	481	175	148	188
	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20
D1	311	315	405	293	117	203	374	189	405	338
D2	484	420	279	356	116	202	423	185	342	207
D3	243	171	149	176	527	455	86	450	225	248
D4	95	194	252	153	158	225	261	207	414	153
D5	134	206	283	170	95	81	274	77	472	261

NB. D denotes depot and R denotes retailer, the same notation is adopted below.

Table 2. Unit transportation risk (the number of affected people) from depot to retailer.

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
D1	148	181	402	396	380	327	354	127	284	338
D2	147	435	260	366	381	426	248	176	460	352
D3	316	437	177	128	180	204	189	407	237	219
D4	301	129	425	289	340	229	437	298	150	207
D5	256	294	618	527	553	481	627	271	403	430
	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20
D1	275	278	339	353	145	202	347	193	399	293
D2	417	337	361	364	146	201	267	431	210	426
D3	250	201	186	294	441	422	144	486	177	253
D4	150	216	256	219	162	207	262	225	425	159
D5	394	442	494	508	150	141	487	137	621	412

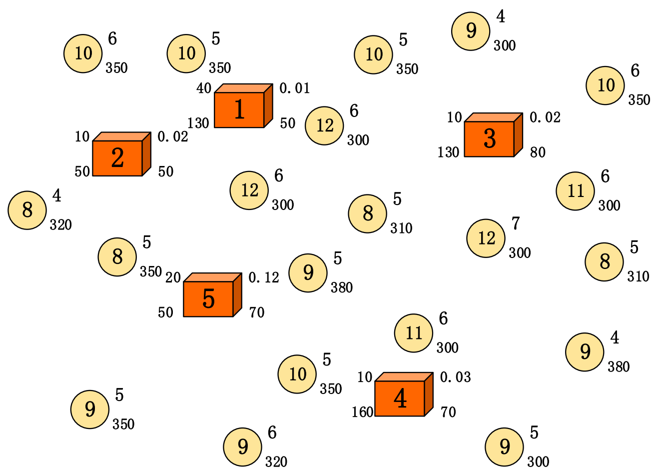

Fig. 3. Location and attributes of candidate depots and retailers.

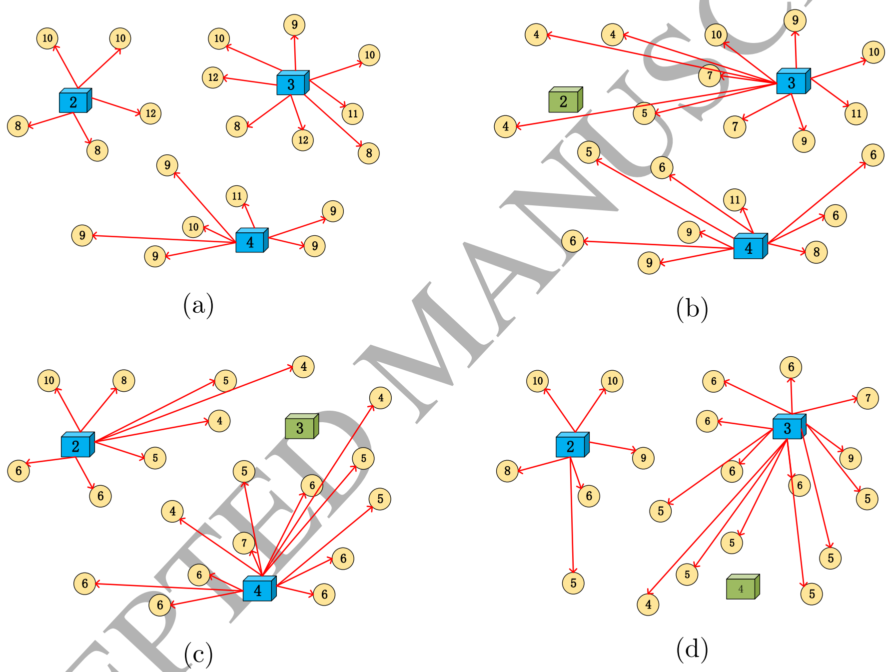

Fig. 4. Location allocation decision and contingency plans.

If we apply the LA model instead of RLA model to this location allocation problem, depots 3, 4 and 5 are determined to rent, and the corresponding amounts of hazmats stored in these depots are 80, 65 and 49 ton, respectively. Since LA model does not take into consideration the scenerio of depot disruption, we can only decide the amount of hazmats stored in each depot and the supply amount to retailer in the scenario where all depots are available. Running the solution method for the LA model, we can obtain the supply amount to retailers in each scenario of depot disruption. Then, we can calculate the minimum systemic risk based on LA by computing the expectation value in all scenarios. The minimum systemic risk is 34584 people, with the systemic cost being 75509 RMB. Both RLA and LA have the solution to rent depot 3 and depot 4. However, depot 2 is chosen by RLA and depot 5 by LA as the remaining one to rent. Comparing these results, we can find that depot 5 is of a higher disruption probability. Comparing with the LA model, the systemic risk of RLA is decreased by 8.72%, and the systemic cost is decreased by 1.25%. We randomly select 20 sets of p_s *which denotes the probability of disruption scenario to implement a statistical test on these two models. The solution* *of RLA model has smaller systemic risk in all these experiments. The maximum rate of systemic risk reduction is 7.39, the minimum rate of systemic risk reduction is 2.12, and the average rate of systemic risk reduction is 4.94. Therefore, for a long-term decision, it is of scientific significance to take reliability into consideration. The RLA model not only reduces risk and cost but also makes contingency plans in advance.*

Table 3. The systmetic risk (the number of affected people) in the statistical test on RLA model and LA model.

NO	probability of disruption scenario	RLA model	LA model	reduction rate
1	0.03 0.03 0.02 0.02 0.10	31572	33814	6.63
2	0.03 0.02 0.04 0.02 0.09	32808	34142	3.91
3	0.06 0.03 0.02 0.03 0.07	31491	32932	4.38
4	0.03 0.03 0.04 0.03 0.07	32310	33868	4.6
5	0.03 0.01 0.02 0.03 0.09	32005	34330	6.77
6	0.01 0.03 0.03 0.02 0.09	32503	34372	5.43
7	0.05 0.01 0.03 0.04 0.09	31812	34351	7.39
8	0.04 0.05 0.03 0.04 0.06	32174	32871	2.12
9	0.05 0.05 0.01 0.01 0.08	31599	32741	3.49
10	0.06 0.01 0.03 0.03 0.07	31545	33573	6.04
11	0.03 0.04 0.03 0.04 0.06	32605	33563	2.85
12	0.04 0.01 0.03 0.04 0.08	31929	34088	6.33
13	0.08 0.01 0.03 0.02 0.06	31278	33058	5.38
14	0.05 0.01 0.05 0.03 0.06	32016	33815	5.32
15	0.03 0.01 0.06 0.04 0.06	32518	33815	3.84
16	0.05 0.01 0.03 0.03 0.08	31545	33836	6.77
17	0.03 0.03 0.03 0.04 0.07	32341	33826	4.39
18	0.04 0.01 0.04 0.04 0.07	32047	34121	6.08
19	0.03 0.02 0.03 0.05 0.07	32343	34078	5.09
20	0.05 0.02 0.03 0.03 0.07	32841	33573	2.18

Example 2. *As mentioned in Section 2, some classical approaches to reliable facility location problem (Snyder and Daskin 2005; Cui et al. 2010) assumped that the depots are uncapacitated and every customer's demand can be satisfied even in the scenario of depot disruption. However, the assumption may be unrealistic, the customer demand usually can not be satisfied in disruption scenarios in real-world practice. Therefore, in this RLA model we add the decision variable* w_{ij}^s*, which denotes the supply amount to customer* j *from depot* i *in scenario* s*, to decrease the impact of supply shortage caused by depot disruption. In addition, hazmats generally need to be stored under a specific temperature and pressure. This could result in different inventory cost between depots since the temperature or pressure control devices are generally different. What's more, the difference of inventory risk between depots should also been captured in the study of location problem. Therefore, we add another decision variable* z_i *which denotes the amount of hazmats stored in depot* i*. Compared with the classical approaches, we have two more decision variables. If these two decision variables are abandoned from the RLA model, we can obtain RLA* model which chooses depot 1, 3, and 4 to rent. The difference with RLA model is that depot 1 instead of depot 2 is determined to rent. Due to the lack of supply shortage consideration, it*

needs to re-optimize the allocation plan when depot disruption causes a supply shortage. After re-optimizing the allocation plan in each disruption scenario, we can obtain the minimum systemic risk based on this solution by calculating the inventory risk and the transportation risk in all disruption scenarios. It is calculated that this solution increases 4.72% systemic risk and 4.16% systemic cost comparing with RLA model. We use the 20 sets of p_s in example 1, and randomly select corresponding 20 sets of h_i which denotes unit inventory risk of depot to implement a statistical test on these two models. The results of this statistical test are listed in Table 4. The solution of RLA model has smaller systemic risk in all these experiments. The maximum rate of systemic risk reduction is 5.54, the minimum rate of systemic risk reduction is 1.34, and the average rate of systemic risk reduction is 2.88. Therefore, RLA model would achieve the better application effect in real-world practice.

Table 4. The systmetic risk (the number of affected people) in the statistical test on RLA model and RLA* model.

NO	unit inventory risk of depot	RLA model	RLA* model	reduction rate
1	20 10 30 30 40	34575	35045	1.34
2	30 50 30 10 10	35252	36232	2.7
3	30 10 30 30 10	34411	35371	2.71
4	30 10 30 30 20	36315	37315	2.68
5	20 10 10 10 50	31309	31799	1.54
6	50 10 20 20 40	33267	35217	5.54
7	30 60 30 20 10	36113	37093	2.64
8	20 10 10 20 30	33827	34307	1.40
9	40 10 20 30 50	33719	34679	2.77
10	40 20 50 10 30	35225	36215	2.73
11	20 30 20 10 10	33873	34363	1.43
12	40 10 30 20 50	34072	35532	4.11
13	30 20 30 20 30	34018	34508	1.42
14	50 40 20 10 10	34211	35701	4.17
15	50 10 20 10 50	34354	36314	5.4
16	30 10 20 10 30	32345	32845	1.52
17	40 50 20 10 10	34136	35606	4.13
18	40 20 50 10 40	35727	36717	2.7
19	30 20 30 10 30	34550	35050	1.43
20	50 40 30 30 10	35983	37943	5.17

Example 3. *Recent studies on reliable facility location problem (An et al. 2014; Alvarez-Miranda et al. 2015) introduced two-stage stochastic programming to address the supply shortage caused by depot disruption as we have posted in Section 2. The two-stage stochastic programming can only ensure the cost induced by supply shortage is within the acceptable range in each disruption scenario. However, the occurrance probability of each disruption scenario has not been considered in the optimization process. In fact, the occurrance probability of each disruption scenario has a direct impact on decision-making of reliable location allocation problem. We can conduct the sensitivity analyses about the occurrance probability of disruption scenario base on the solution of RLA model. Analyzing the comparison result of RLA and LA model, it shows that depot 2 with lower disruption*

probability was determined to rent by RLA model and depot 5 with higher disruption probability was determined to rent by LA model. We increase the value of p_2 while decreasing the value of p_5. When p_2 is 0.08 and p_5 is 0.07, RLA model determines to rent depot 5 instead of depot 2. It demonstrates that the probability of each disruption scenario does have a direct impact on decision-making. Therefore, we prefer to formulate the RLA model instead of a two-stage stochastic programming to address the reliable location allocation problem.

5 Case studies

In this section, a real-world case study on hazmats logistics is presented to evaluate the performance of the proposed model. The experiment involves a company offering six candidate depots and fifty retailers needing to rent certain depots from the candidate depot set and to design an allocation plan. The goal is to minimize risk under a given budget constraint. Since the depots are subject to independent disruption with a different probability, it is essential to take reliability into account so as to minimize the systemic risk and make contingency plans for depot disruption in advance.

According to the coordinates of latitude and longitude, we mark the locations of the candidate depots and the retailers as shown in Figure 5, where the red circles denote the depots and the yellow stars denote the retailers. The longitude and latitude coordinates of these depots and retailers are listed in Table 5 and Table 6, respectively. Transport routes are specified with regards to the risk minimization principle while complying with the rule of hazmats vehicle prohibition in Beijing (where the example data is taken from). The distances are calculated by Baidu Maps. The unit transportation cost from a depot to a retailer is given in Table 7, and the unit transportation risk is given in Table 8. The maximum capacity, rental cost, disruption probability, unit inventory risk, and unit inventory cost of the depots are listed in Table 9, and the regular demand, minimum demand, and shortage penalty coefficient of retailers are listed in Table 10.

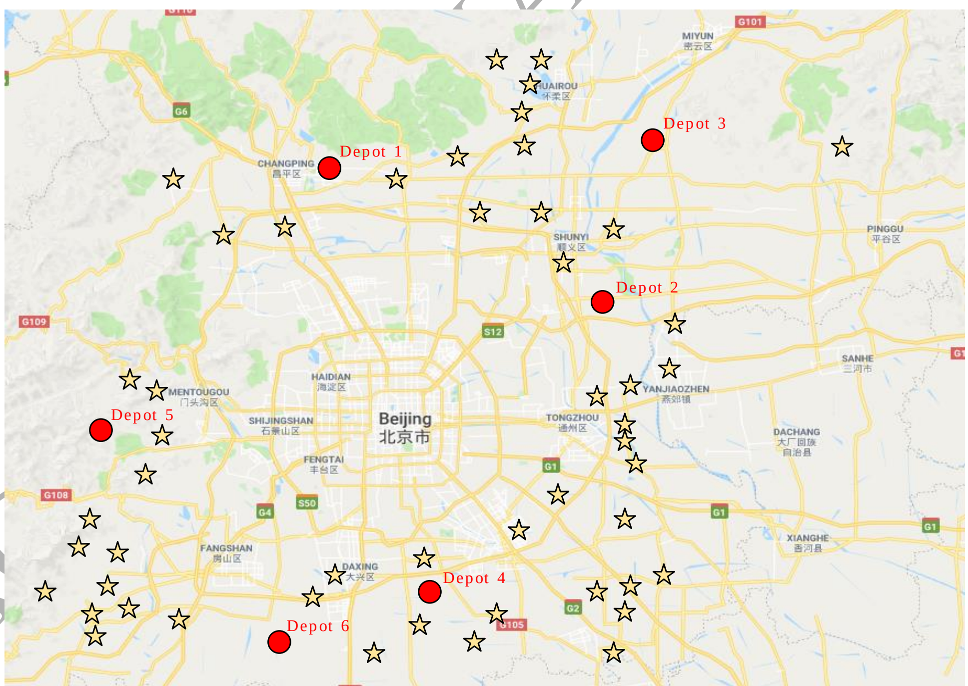

Fig. 5. Mapflag.

In this experiment, the total budget G is the value that is increased from the minimum systemic cost without risk consideration by a certain percentage which can be accepted by the company. Generally speaking, increasing the total budget will reduce the risk. However, an increase in the total budget will lead to a decline in the competitiveness of an company. Therefore, it is necessary to analyze the impact of the total budget on

Table 5. Longitude and latitude coordinates of depots.

	D1	D2	D3	D4	D5	D6
Longitude	116.357904	116.741790	116.727850	116.532978	116.073176	116.297603
Latitude	40.240594	40.047901	40.244421	39.727549	39.898783	39.675097

systemic risk. The relationship between the systemic risk and the total budget is shown in Figure 6. When the value of which the minimum systemic cost without risk consideration is increased by 5 percent is taken as the total budget G, the systemic risk affects 44387 people. When the minimum systemic cost increases by 10 precent, the systemic risk drops to 40503 people. By comparing these two points, we can see that a 5 percent increase in the total budget can lead to nearly 9 percent risk reduction. After these two points, the systemic risk is decreased twice with the total budget increasing. However, the systemic risk reduction is very limited. When the increased percentage of minimum systemic cost is 30 percent, the systemic risk is 39637 people. When the minimum systemic cost increases by 60 precent, the systemic risk falls to 39487 people. In this numerical experiment, the company agrees to raise minimum systemic cost by up to 10 percent in order to reduce the systemic risk. Considering the characteristic of hazmats, the value that the minimum systemic cost is increased by 10 percent is taken as the total budget G.

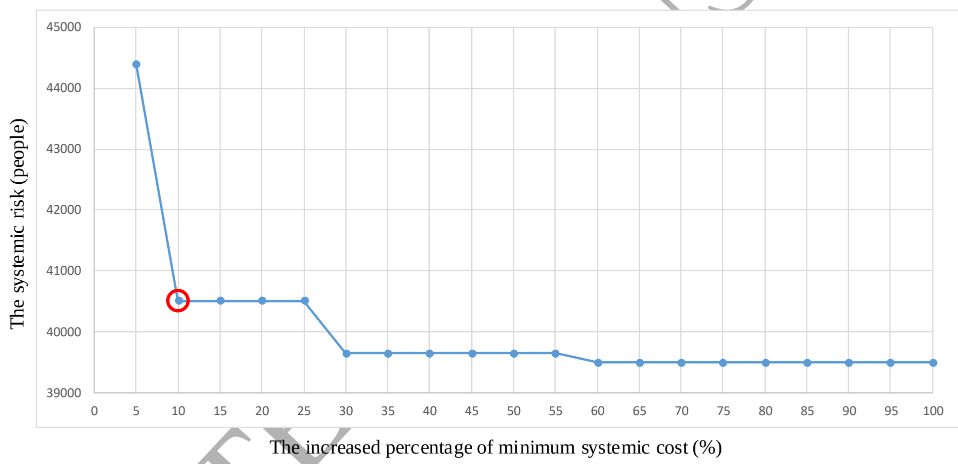

Fig. 6. Systemic risk with respect to total budget.

By executing the Lingo software with a personal computer of the following specification: Intel i3-3240 CPU, 3.40 GHz, 4GB RAM, running Windows 10, the following results of RLA are obtained. The computation time is 52sec. Depots 2, 4 and 5 are determined to rent, and the corresponding amounts of hazmats stored in these depots are 90, 99 and 72 ton, respectively. The systemic risk involves 40503 people, which means that 40503 people would be affected if hazmats incident incurred, with the systemic cost being 227891 RMB. In the scenario of all depots being available, every retailer's regular demand can be satisfied. The allocation scheme for depots and retailers is shown in Table 11. In particular, if depot 2 is disrupted, the contingency plan is shown in Table 12. If depot 4 is disrupted, the contingency plan is shown in Table 13. If however, depot 5 is disrupted, the contingency plan is shown in Table 14.

If we choose to use the LA model instead, depots 3, 4 and 5 are determined to rent, and the corresponding amounts of hazmats stored in these depots are 89, 100 and 72 ton, respectively. The computation time is 5sec. Through the use of LA, we can only decide the amount of hazmats stored in each depot and the supply amount to a retailer in the scenario where all depots are available. Running the solution method for the LA model, the supply amount to retailers from depots in each scenario of depot disruption can be obtained. Then, we can calculate the minimum systemic risk based on LA by computing the expectation value in all scenarios. The

Table 6. Longitude and latitude coordinates of retailers.

	R1	R2	R3	R4	R5
Longitude	116.243486	116.610601	116.022579	116.41747	116.43122
Latitude	40.07173	40.149935	39.711597	39.634003	39.768911
	R6	R7	R8	R9	R10
Longitude	116.602817	115.990608	116.153624	116.739589	116.740079
Latitude	39.690718	39.694297	40.23577	39.869763	39.809815
	R11	R12	R13	R14	R15
Longitude	116.723401	116.553442	116.573539	116.567869	116.572881
Latitude	39.915897	39.824201	39.658351	39.851397	40.230372
	R16	R17	R18	R19	R20
Longitude	116.694993	116.440147	116.653724	116.027083	116.357112
Latitude	40.370577	39.742579	40.333048	39.712659	39.796029
	R21	R22	R23	R24	R25
Longitude	116.251519	117.156788	116.751455	115.964906	116.803321
Latitude	40.211196	40.168711	40.145688	39.683258	39.714735
	R26	R27	R28	R29	R30
Longitude	116.590813	116.636481	116.488741	116.050006	116.002835
Latitude	39.811158	40.346429	39.626629	39.729663	39.703621
	R31	R32	R33	R34	R35
Longitude	116.740153	116.114598	116.660348	116.864187	116.823992
Latitude	39.963298	39.932347	40.253746	39.729759	39.685087
	R36	R37	R38	R39	R40
Longitude	116.56635	116.662083	116.810158	116.799731	116.693243
Latitude	39.814913	39.924163	39.725348	40.058215	40.03421
	R41	R42	R43	R44	R45
Longitude	115.959544	116.669112	116.722595	116.014377	116.524274
Latitude	39.767891	40.111687	39.887608	39.708974	39.68667
	R46	R47	R48	R49	R50
Longitude	115.997165	115.99906	116.109351	116.836854	116.665285
Latitude	39.739807	39.731235	39.951469	39.816523	40.286712

Table 7. Unit transportation cost (RMB) from depot to retailer.

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
D1	270	380	910	890	740	860	950	200	720	700
D2	510	240	920	670	540	570	960	700	310	400
D3	560	190	1260	960	750	810	1290	610	550	640
D4	610	710	490	110	150	110	520	970	370	310
D5	460	790	410	560	510	760	370	550	740	680
D6	760	1160	300	140	210	390	340	860	500	440

	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20
D1	690	700	900	650	260	450	830	420	900	750
D2	210	430	560	340	350	500	580	460	920	340
D3	520	680	850	600	210	180	830	170	1270	580
D4	400	220	110	260	820	970	150	930	480	420
D5	730	630	710	620	790	980	500	940	410	760
D6	540	380	330	390	1170	1010	190	1320	300	550

	R21	R22	R23	R24	R25	R26	R27	R28	R29	R30
D1	240	780	450	1000	990	700	440	920	880	950
D2	480	530	150	1010	560	440	480	710	1170	960
D3	500	440	150	1340	980	680	200	960	1240	1300
D4	760	1100	740	570	350	230	960	150	480	530
D5	510	1270	830	430	910	640	970	690	370	360
D6	630	1280	870	390	630	360	1000	320	290	340

	R31	R32	R33	R34	R35	R36	R37	R38	R39	R40
D1	640	490	380	970	980	690	620	950	640	520
D2	140	680	370	450	490	440	190	450	130	640
D3	470	920	270	770	900	710	410	780	290	990
D4	460	640	850	400	360	220	470	450	690	590
D5	850	260	910	960	880	620	710	940	900	240
D6	590	510	1290	690	620	350	600	580	820	540

	R41	R42	R43	R44	R45	R46	R47	R48	R49	R50
D1	800	420	720	920	870	800	790	490	870	370
D2	1000	190	240	930	630	1200	930	680	460	400
D3	1270	230	550	1270	870	1240	1220	920	660	150
D4	570	670	390	490	150	520	510	630	440	880
D5	280	750	740	420	640	300	310	160	880	900
D6	390	800	520	310	270	340	330	510	660	1280

Table 8. Unit transportation risk (the number of affected people) from depot to retailer.

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
D1	168	201	422	416	400	347	374	147	304	358
D2	301	129	425	389	340	229	437	298	259	207
D3	256	144	468	377	403	331	477	271	253	280
D4	381	301	234	119	161	119	244	440	198	180
D5	225	385	210	316	331	376	198	253	420	352
D6	316	437	177	128	180	204	189	407	237	219

	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20
D1	295	298	359	373	165	222	367	213	419	313
D2	150	216	256	219	162	207	362	225	425	259
D3	244	292	344	358	150	141	337	137	471	262
D4	207	153	119	255	334	380	116	368	231	213
D5	417	337	361	364	385	443	267	431	210	426
D6	250	201	186	294	441	422	144	486	177	253

	R21	R22	R23	R24	R25	R26	R27	R28	R29	R30
D1	159	322	222	449	446	298	219	425	413	434
D2	280	247	161	452	256	219	211	381	441	437
D3	237	219	131	492	383	292	147	377	462	480
D4	376	419	310	259	192	156	377	131	231	247
D5	301	531	397	216	422	340	440	355	198	195
D6	337	534	350	204	277	195	449	183	174	189

	R31	R32	R33	R34	R35	R36	R37	R38	R39	R40
D1	280	234	201	380	443	355	274	374	280	244
D2	128	352	168	222	234	219	244	222	215	340
D3	228	365	137	319	359	301	210	322	174	386
D4	225	340	344	207	195	153	228	222	295	325
D5	404	128	422	487	463	334	411	481	419	159
D6	265	241	477	295	274	192	268	262	334	250

	R41	R42	R43	R44	R45	R46	R47	R48	R49	R50
D1	328	213	304	425	410	328	325	234	350	198
D2	449	144	139	428	277	450	428	352	325	190
D3	471	156	253	471	350	462	456	365	286	128
D4	259	289	204	234	98	244	241	337	219	353
D5	171	373	420	213	340	177	180	134	463	419
D6	204	328	244	180	168	189	186	241	286	474

Table 9. Attributes of depots including maximum capacity (ton), rental cost (RMB), disruption probability, unit inventory risk (number of affected people), and unit inventory cost (RMB).

	Maximum capacity	Rental cost	Disruption probability	Unit inventory risk	Unit inventory cost
D1	100	50000	0.01	10	13
D2	90	45000	0.02	8	14
D3	90	35000	0.1	8	10
D4	100	50000	0.03	9	13
D5	80	40000	0.03	8	14
D6	100	50000	0.01	9	14

Table 10. Attributes of retailers including regular demand (ton), minimum demand (ton), and shortage penalty coefficient.

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
Regular demand	4	5	6	5	6	9	4	6	7	4
Minimum demand	1	2	2	2	2	5	1	2	2	1
Shortage penalty coefficient	320	350	300	300	350	300	310	350	300	380
	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20
Regular demand	5	6	8	5	6	9	4	5	6	5
Minimum demand	1	2	3	2	2	4	1	2	2	2
Shortage penalty coefficient	320	350	300	300	350	300	310	350	300	380
	R21	R22	R23	R24	R25	R26	R27	R28	R29	R30
Regular demand	4	3	6	4	8	9	4	5	6	5
Minimum demand	1	1	2	1	2	3	1	1	2	1
Shortage penalty coefficient	320	350	300	300	350	300	310	350	300	380
	R31	R32	R33	R34	R35	R36	R37	R38	R39	R40
Regular demand	3	5	6	3	5	8	3	3	6	4
Minimum demand	1	2	2	1	2	3	1	1	2	1
Shortage penalty coefficient	320	350	300	300	350	300	310	350	300	380
	R41	R42	R43	R44	R45	R46	R47	R48	R49	R50
Regular demand	3	3	6	4	5	5	4	6	6	4
Minimum demand	1	1	2	1	1	2	1	2	2	1
Shortage penalty coefficient	320	350	300	300	350	300	310	350	300	380

Table 11. Allocation scheme for depots and retailers.

D2	R2 R9 R11 R15 R16 R18 R20 R21 R22 R23 R27 R31 R33 R37 R39 R42 R43 R50
D4	R4 R5 R6 R10 R12 R13 R14 R17 R25 R26 R28 R34 R35 R36 R38 R45 R49
D5	R1 R3 R7 R8 R19 R24 R29 R30 R32 R40 R41 R44 R46 R47 R48

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Table 12. contingency plan in scenario of depot 2 disruption.

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
D4	-	2	-	5	6	9	-	-	2	1
D5	1	-	6	-	-	-	4	2	-	-

	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20
D4	1	6	8	2	-	-	4	2	-	2
D5	-	-	-	-	2	4	-	-	6	-

	R21	R22	R23	R24	R25	R26	R27	R28	R29	R30
D4	-	1	2	-	2	9	1	5	-	-
D5	1	-	-	4	-	-	-	-	6	5

	R31	R32	R33	R34	R35	R36	R37	R38	R39	R40
D4	1	-	2	1	2	8	1	1	2	-
D5	-	5	-	-	-	-	-	-	-	4

	R41	R42	R43	R44	R45	R46	R47	R48	R49	R50
D4	-	1	2	-	5	-	-	-	2	1
D5	3	-	-	4	-	5	4	6	-	-

Table 13. contingency plan in scenario of depot 4 disruption.

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
D2	-	5	-	-	-	5	-	-	7	1
D5	1	-	6	2	2	-	4	2	-	-

	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20
D2	5	2	3	2	2	4	-	2	-	5
D5	-	-	-	-	-	-	1	-	6	-

	R21	R22	R23	R24	R25	R26	R27	R28	R29	R30
D2	-	1	6	-	2	3	1	-	-	-
D5	1	-	-	4	-	-	-	1	6	5

	R31	R32	R33	R34	R35	R36	R37	R38	R39	R40
D2	3	-	2	1	2	3	3	1	6	-
D5	-	5	-	-	-	-	-	-	-	4

	R41	R42	R43	R44	R45	R46	R47	R48	R49	R50
D2	-	3	6	-	1	-	-	-	2	1
D5	3	-	-	4	-	5	4	6	-	-

Table 14. contingency plan in scenario of depot 5 disruption.

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
D2	1	5	-	-	-	-	-	2	7	-
D4	-	-	2	5	6	9	1	-	-	4
	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20
D2	5	-	-	2	6	9	-	2	-	5
D4	-	6	8	-	-	-	4	-	2	-
	R21	R22	R23	R24	R25	R26	R27	R28	R29	R30
D2	1	1	6	-	-	-	1	-	-	-
D4	-	-	-	1	8	9	-	5	2	1
	R31	R32	R33	R34	R35	R36	R37	R38	R39	R40
D2	3	2	6	1	-	-	3	1	6	-
D4	-	-	-	-	5	8	-	-	-	1
	R41	R42	R43	R44	R45	R46	R47	R48	R49	R50
D2	-	3	6	-	-	-	-	-	2	4
D4	1	-	-	1	5	2	1	2	-	-

minimum systemic risk is 44183 people, with the systemic cost being 232352 RMB.

Both RLA and LA have the solution to rent depot 4 and depot 5. However, depot 2 is chosen by RLA and depot 3 by LA as the remaining one to rent. Comparing these results, we can find that these two depots have the same maximum capacity, and that depot 3 has a lower cost, but is of a higher disruption probability. Moreover, when depot 4 or 5 is disrupted, depot 3 will lead to more transportation risk and cost than depot 2. Since the RLA model fully considers reliability, it selects depot 2. Obviously, from a long-term decision viewpoint, the consideration of reliability in advance helps reduce systemic risk and cost. Comparing to the LA model, the systemic risk of RLA is decreased by 8.33%, and the systemic cost is decreased by 1.92%. Furthermore, the RLA model not only guarantees the minimum demand to every retailer in the scenarios of depot disruption, but also makes contingency plans in advance.

6 Conclusions and further research

In this paper, we proposed the problem of reliable location allocation for hazmats, considering that the depots are subject to the risk of disruption that may be caused by many factors. A difference of our study with other literatures is that we focus on minimizing the systemic risk instead of transportation risk under given budget constraints. This is because location allocation, as a decision with mid-term and long-term effects, would be affected by both storage risk and depots disruption. The aim for such a novel development has been to answer the following questions for company decision makers: how to determine the optimal depot locations, how many hazmats would be stored in each located depot, and what is the optimal allocation (transportation) plans. Furthermore, contingency plan-making has contributed to minimizing the risks and economic losses caused by depot disruption.

Numerical experimental investigation has been carried out to illustrate the effectiveness of the model. Comparison experiment of RLA model and LA model reveals the necessity and importance of taking reliability into account. Then the comparison with other similar approaches to reliable facility location problem shows the superiority of RLA model. What's more, we implement two statistical tests to illustrate the stability of our proposed model. Finally, a real-world case study is provided in Section 5. The results have demonstrated that the proposed modeling method not only decreased the systemic risk by 8.33% but also decreased the systemic cost by 1.92%. It has been shown that if the reliability and contingency plan was considered, depots with lower disruption probability would be more likely to be selected. This is because the disruption probability of depot has impact on both the systemic risk and the systemic cost in a long-term decision. Some management insights for the company are obtained. For a mid-term and long-term decision, it is of necessity and importance to take reliability into account. The company decision makers should jointly optimize the location, allocation and the contingency plans by a combined consideration of depot availability and disruption. Only in this way, the decision-making can achieve the anticipated effect.

A number of interesting research topics motivated by the present work would be worth further investigating. First, it would be appealing to develop a more detailed model by taking more actual factors such as uncertain demand of customer, temporary depot, traffic restrictions and special-line transportation into consideration. Second, for the convenience of management, this study presumes that each customer can only be serviced by one depot. In certain real life scenarios, reliable location allocation with each customer being serviced by multiple depots is worthy of further investigation. Third, only static decision rules are considered in this study, ignoring the duration and the frequency of the depot disruptions. Incorporating these factors into our model would allow us to examine optimal decision rules in a dynamic environment. Finally, all constraints are currently regarded as being of the same significance. It would be interesting to investigate whether data-driven weighting schemes such as the techniques introduced in Li et al. (2018) may be adapted to distinguish the contributions of different constraints.

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